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Gauge Symmetries of Electroweak Interactions

S. Rajpoot[1]

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By considering the symmetries associated with baryon number and lepton number conservation as gauge symmetries, the underlying gauge symmetry of weak electromagnetic interactions is shown to be $SU(2)_L \times U(1) \times U(1)_{\text{Baryon}} \times U(1)_{\text{Lepton}}$. If right-handed currents exist on a par with the observed left-handed ones, then the full symmetry of electroweak interactions that emerges is $SU(2)_L \times SU(2)_R \times U(1)_{\text{Baryon}} \times U(1)_{\text{Lepton}}$. These symmetries offer a rich spectrum of massive neutral gauge bosons, one of which is the massive neutral boson of the standard $SU(2)_L \times U(1)_Y$ model.

That nature is in favor of the gauge principle (Yang and Mills, 1954) is demonstrated well by the discovery of the massive gauge particles of the standard $SU(2)_L \times U(1)_Y$ model (Glashow, 1961; Weinberg, 1967; Salam, 1968) of weak and electromagnetic interactions. However, the issue of the weak hypercharge (Rajpoot, 1986) may well be indicative of the fact that the $SU(2)_L \times U(1)_Y$ gauge symmetry is only an effective residual gauge symmetry that has been probed at the presently attained low energies.

Nature does provide partial clues to the possible structure of the weak and electromagnetic interactions.

Experiments indicate that the baryon and lepton numbers associated with conventional quarks and leptons are conserved by the strong and the electroweak interactions. In analogy with electric charge, it is tempting to identify the conserved currents associated with baryon and lepton numbers as coupled to photonlike gauge particles. The idea is a straightforward generalization of the old idea (Wigner, 1952) of associating baryon number conservation with an Abelian gauge symmetry. However, the symmetries associated with baryon and lepton numbers must be broken, since no photonlike gauge particles have been observed.

[1]Department of Physics, University of California, Riverside, California 92521.

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Conventional quarks are assigned baryon number equal to 1/3 and conventional leptons are assigned lepton number equal to one. From this assignment it follows that in the case of the first generation of quarks and leptons, the currents associated with baryon number and lepton number are

$$\begin{aligned} J^{\mu}_{\text{baryon}} &= \tfrac{1}{3}(\bar{u}_{iL}\gamma^{\mu}u_{iL} + \bar{d}_{iL}\gamma^{\mu}d_{iL} + \bar{u}_{iR}\gamma^{\mu}u_{iR} + \bar{d}_{iR}\gamma^{\mu}d_{iR}) \\ &= \tfrac{1}{3}(\bar{u}_i\gamma^{\mu}u_i + \bar{d}_i\gamma^{\mu}d_i) \end{aligned} \tag{1}$$

and

$$\begin{aligned} J^{\mu}_{\text{lepton}} &= \bar{e}_L\gamma^{\mu}e_L + \bar{e}_R\gamma^{\mu}e_R + \bar{\nu}_{eL}\gamma^{\mu}\nu_{eL} \\ &= \bar{e}\gamma^{\mu}e + \bar{\nu}_{eL}\gamma^{\mu}\nu_{eL} \end{aligned} \tag{2}$$

where $i = 1, 2, 3$ for the three colors and the chiral fields are defined to be $\psi_L = \frac{1}{2}(1+\gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1-\gamma_5)\psi$. Note that at this stage the neutrino has no right-handed counterpart. Since the electric charges of these particles are known, the electromagnetic current in units of the proton charge e is

$$J^{\mu}_{\text{em}} = \tfrac{2}{3}\bar{u}_i\gamma^{\mu}u_i - \tfrac{1}{3}\bar{d}_i\gamma^{\mu}d_i - \bar{e}\gamma^{\mu}e \tag{3}$$

Further, weak interactions require that the following charged left-handed weak isospin currents also be present:

$$J^{\mu}_{+L} = \frac{1}{\sqrt{2}}\bar{e}_L\gamma^{\mu}\nu_{eL} + \frac{1}{\sqrt{2}}\bar{d}_{iL}\gamma^{\mu}u_{iL} \tag{4}$$

$$J^{\mu}_{-L} = (J^{\mu}_{+L})^{\dagger} = \frac{1}{\sqrt{2}}\bar{\nu}_{eL}\gamma^{\mu}e_L + \frac{1}{\sqrt{2}}\bar{u}_{iL}\gamma^{\mu}d_{iL} \tag{5}$$

where the dagger denotes Hermitian conjugate. The Cabibbo angle is suppressed for convenience, although it is not necessary to do so. The neutral weak isospin current is taken to be

$$J^{\mu}_{0L} = \tfrac{1}{2}(\bar{u}_{iL}\gamma_{\mu}u_{iL} - \bar{d}_{iL}\gamma^{\mu}d_{iL}) + \tfrac{1}{2}(\bar{\nu}_{eL}\gamma^{\mu}\nu_{eL} - \bar{e}_L\gamma^{\mu}e_L) \tag{6}$$

A closer examination of the electromagnetic current J^{μ}_{em} in equation (3) reveals the following underlying structure:

$$J^{\mu}_{\text{em}} = J^{\mu}_{0L} + J^{\mu}_{\text{baryon}} - J^{\mu}_{\text{lepton}} + J^{\mu}_{0R} \tag{7}$$

where J^{μ}_{0R} consists of only right-handed quarks and leptons,

$$J^{\mu}_{0R} = \tfrac{1}{2}\bar{u}_{iR}\gamma^{\mu}u_{iR} - \tfrac{1}{2}\bar{d}_{iR}\gamma^{\mu}d_{iR} - \tfrac{1}{2}\bar{e}_R\gamma^{\mu}e_R \tag{8}$$

The currents $J^{\mu}_{-L}, J^{\mu}_{+L}, J^{\mu}_{0L}$ form the celebrated $SU(2)_L$ triplet of the standard $SU(2)_L \times U(1)_Y$ model, while J^{μ}_{0R}, J^{μ}_{baryon}, and J^{μ}_{lepton} correspond to the currents coupled to the additional Abelian symmetries. Thus, the symmetry of weak and electromagnetic interactions is extended from $SU(2)_L \times U(1)_Y$ to $SU(2)_L \times U(1)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$.

Under this new symmetry of weak and electromagnetic interactions, the quarks and leptons of the first generation are still left-handed doublets and right-handed singlets, but with different transformation properties,

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \sim (2, 0, \tfrac{1}{3}, 0) \tag{9}$$

$$u_{iR} \sim (1, \tfrac{1}{2}, \tfrac{1}{3}, 0) \tag{10}$$

$$d_{iR} \sim (1, -\tfrac{1}{2}, \tfrac{1}{3}, 0) \tag{11}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim (2, 0, 0, 1) \tag{12}$$

$$e_R \sim (1, -\tfrac{1}{2}, 0, 1) \tag{13}$$

Let T_L, T_R, T_{baryon}, and T_{lepton} denote the generators of the currents J^{μ}_{0L}, J^{μ}_{0R}, J^{μ}_{baryon}, and J^{μ}_{lepton}. The generator Q of the electromagnetic current is the following linear combination of T_L, T_R, T_{baryon}, and T_{lepton}:

$$Q = T^0_L + T^0_R + \tfrac{1}{2} T^0_{\text{baryon}} - \tfrac{1}{2} T^0_{\text{lepton}} \tag{14}$$

Comparing this expression with the conventional formula for the electric charge operator in the standard $SU(2)_L \times U(1)_Y$ model,

$$Q = T_L + \tfrac{1}{2} Y \tag{15}$$

one identifies the hypercharge generator Y with the following combination of T_R, T_{baryon}, and T_{lepton}:

$$\tfrac{1}{2} Y = T^0_R + \tfrac{1}{2} T^0_{\text{baryon}} - \tfrac{1}{2} T^0_{\text{lepton}} \tag{16}$$

Thus, the mysterious hypercharge of the standard $SU(2)_L \times U(1)_Y$ model is nothing but a linear combination of the generators of lepton number, baryon number, and an additional $U(1)$ generator that couples only to right-handed quarks and leptons.

The structure of the theory can be made more symmetric by adding an additional lepton (E^0) that is electrically neutral but carries half a unit of T_R and one unit of lepton number. The neutral lepton of this stage need not be the right-handed counterpart of the left-handed neutrino ν_e. However, the structure of J_{0R} now resembles that of J_{0L} and it is tempting to identify the generator of J_{0R} with that of right-handed weak isospin. If it is assumed that E^0 is the right-handed neutrino and that nature is symmetric between left and right-handed weak interactions, then right-handed charged currents are required to exist on par with the left-handed ones,

$$J^{\mu}_{+R} = \frac{1}{\sqrt{2}} \bar{e}_R \gamma^{\mu} \nu_{eR} + \frac{1}{\sqrt{2}} \bar{d}_{iR} \gamma^{\mu} u_{iR} \tag{17}$$

$$J^{\mu}_{-R} = \frac{1}{\sqrt{2}} \bar{\nu}_{eR} \gamma^{\mu} e_R + \frac{1}{\sqrt{2}} \bar{u}_{iR} \gamma^{\mu} d_{iR} \tag{18}$$

The currents $J^{\mu}_{-R}, J^{\mu}_{+R}$ together with J^{μ}_{0R} form another $SU(2)$ and the underlying symmetry of the electroweak interaction is extended from $SU(2)_L \times U(1)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ to $SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$. In this case J_{0R} is the current associated with the right-handed weak isospin generator T_R. Under the master symmetry $SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$, the quarks and leptons of the first generation are organized as follows:

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \sim (2, 1, \tfrac{1}{3}, 0) \tag{19}$$

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_R \sim (1, 2, \tfrac{1}{3}, 0) \tag{20}$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \sim (2, 1, 0, 1) \tag{21}$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_R \sim (1, 2, 0, 1) \tag{22}$$

We have similar results for the members of the second and the third generations, assuming that the top quark exists. There are anomalies associated with the currents of the symmetry $SU(2)_L \times U(1)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ as well as the master symmetry $SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$. The simplest way to cancel these anomalies is to add mirror fermions. These fermions transform in the same way as the conventional fermions [equations (9)-(22)], but carry opposite chiralities.

The gauge symmetries $SU(2)_L \times SU(2)_R \times U(1)$ (Pati and Salam, 1974) and $SU(2)_L \times U(1)_Y$ are spontaneously broken residual symmetries of the master gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$. The $U(1)$ in the residual symmetry $SU(2)_L \times SU(2)_R \times U(1)$ corresponds to the familiar baryon minus lepton number generator (Mohapatra and Marshak (1980a,b).

The rich structure of the $SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ electroweak symmetry offers the exciting possibility of producing two extra massive neutral gauge bosons beyond the mass scale of the standard neutral boson of the $SU(2)_L \times U(1)_Y$ model and right-handed charged bosons in the 200–1000 GeV energy range on the assumption that the massive bosons are "light." Models of electroweak interaction based on the $SU(2)_L \times U(1)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ and $SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ are presently under study (Rajpoot, in preparation).

Finally, it is straightforward to embed the electroweak symmetries $SU(2)_L \times U(1)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ or $SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ in a grand unifying symmetry. For conventional gluons of strong interactions belonging to vectorial $SU(3)$ of color, the unifying symmetry is required to have rank ≥ 6. For chiral color (Pati and Salam, 1975; for review see Frampton and Glashow, 1987), the rank of the unifying gauge symmetry is required to be ≥ 8. Some possible candidates containing $SU(3)_{L+R} \times SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ are $SO(18)$, $SU(4)^3$ with discrete symmetries for one coupling constant, and $SU(16)$ (Pati *et al.*, 1975, 1981). Possible candidates containing chiral color $SU(3)_L \times SU(3)_R \times SU(2)_L \times SU(2)_R \times U(1)_{\text{baryon}} \times U(1)_{\text{lepton}}$ are $SU(16)$, $SU(4)^4$, and $SO(12)_L \times SO(12)_R$. These symmetries contain a rich spectrum of exotic quarks and leptons as well as gauge bosons, some of which could lie in the energy range of the Tevatron and the accelerators of the 1990s.

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